# Molecular Interaction of Water Vapor and Air 

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#### Abstract

The factor $f_{w}(T, p)$ by which air increases the saturation vapor pressure of water has been measured accurately for temperatures of 20,50 , and $75^{\circ} \mathrm{C}$ and pressures to 14 MPa . An account is given of the measurements and of the analysis of the results to yield the interaction second virial coefficient $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ and the first experimental values of the interaction third virial coefficient $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ of the vapor-gas mixture. When a Lennard-J ones potential is fitted, the three values of $\mathrm{B}_{\mathrm{aw}}$ are found to be highly mutually consistent, allowing that coefficient and its derivatives to be calculated for a wide range of temperatures. The three values of $\mathrm{C}_{\text {aaw }}$ define the magnitude of this coefficient well for temperatures from about 0 to $100{ }^{\circ} \mathrm{C}$, but for want of a reliable theoretical interpretation yield little information concerning its derivatives. The results for $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ are compared with published experimental values, and those for both $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ and $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ with published theoretical estimates. The work allows the vapor-pressure enhancement factor $f_{w}(T, p)$ and the vapor-concentration enhancement factor $g_{w}(T, p)$ to be calculated for a wide range of conditions, and substantially improves the accuracy with which the thermodynamic properties of the water-air system can be calculated.


## Introduction

Vapor-Concentration and Vapor-Pressure Enhancement Factors for Saturated Water Vapor in Air. The presence of air at an elevated pressure substantially increases the concentration of water vapor in equilibrium with liquid water or ice as if the condensed phase were soluble in air. Components of the effect arise from the intermol ecular forces in the gas mixture, the action of the pressure on the condensed phase (the Poynting effect), and the vapor-pressure lowering associated with the air dissol ved in that phase (the Raoult effect). F or a temperature of $20^{\circ} \mathrm{C}$ and a pressure of 15 MPa , for example, the approximate contributions of these components are, respectively, an increase of $39 \%$, an increase of $12 \%$, and a decrease of $0.2 \%$, while the whole effet is an increase of approximately 55\%.

The vapor-concentration enhancement factor $g_{w}(T, p)$ may be defined by

$$
\begin{equation*}
c_{w}=g_{w}(T, p) c_{w}^{0} \tag{1}
\end{equation*}
$$

where $c_{w}$ is the saturation concentration in moles per unit volume of water vapor in air at the temperatureT and total pressure $p$ and $c_{w}^{0}$ that of the pure vapor at the same temperature. Noting that the partial pressure $p_{w}$ of the vapor is the product of the vapor mole fraction $x_{w}$ and the total pressure, we may also define the vapor-pressure enhancement factor $f_{w}(T, p)$ by

$$
\begin{equation*}
p_{w}=f_{w}(T, p) p_{w}^{o} \tag{2}
\end{equation*}
$$

where $p_{w}^{o}$ is the saturation vapor pressure of pure water at $T$. Equation 2 is equivalent to

$$
\begin{equation*}
x_{w}=f_{w} p_{w}^{o} / p \tag{3}
\end{equation*}
$$

It can easily be shown that

$$
\begin{equation*}
f_{w}=\left(Z / Z_{w}^{0}\right) g_{w} \tag{4}
\end{equation*}
$$

where $Z_{w}^{0}$ is the compressibility factor of the pure saturated vapor, and $Z$ that of the saturated mixture. At
elevated pressures the departures of $g_{w}$ and $f_{w}$ from unity are much greater than those of $Z_{w}^{0}$ and $Z$, and so, in view of eq 4, are roughly equal. For instance, in the above numerical example, $Z_{w}^{0}=0.999, Z=1.001, g_{w}=1.549$, and $f_{w}=1.552$. Analogous enhancement factors $g_{i}$ and $f_{i}$ may be defined for air in equilibrium with ice.

The factor $f_{w}$ is much more amenable to direct measurement and of much wider practical application than $\mathrm{g}_{\mathrm{w}}$. The reason is that $f_{w}$ relates directly to the composition of the gas mixture, which remains constant when the gas flows through a system in which the temperature and pressure vary. Some direct applications of $f_{w}(T, p)$ are to the calculation of the composition of moist air from a humidity generator in which the gas is saturated over the liquid at a known temperature and pressure, the calculation of the temperature at which compressed air of a given water content will deposit liquid in tubing through which it flows, and the deduction of the composition of moist air from dewpoint measurements made at an elevated pressure.

To measure $f_{w}$, a continuous stream of air may be saturated over liquid water at a measured temperature $T$ and total pressure p and then expanded to a lower pressure at which its composition may be determined. Then $f_{w}$ may be calculated from eq 3. The general method has been reviewed by Rowlinson and Richardson (1959) in the context of volatile solids, and by Mason and Spurling (1969).

Use of Virial and Interaction Virial Coefficients. We derive below an equation which expresses $\ln f_{w}$ as the sum of three parts which represent the intermolecular forces, the Poynting effect, and the Raoult effect. To obtain the first part, an expression for the partial thermodynamic potential of the water vapor in the gas mixture is obtained from the virial form of the equation of statefor the mixture. This form is chosen because it has an exact statistical mechanical basis.

The general form of the virial equation of state has been derived by Mayer (1939). It can easily be shown to imply that, when one of the gases present is itself a mixture of constant composition, it is exact to treat that gas as a single component, not only in the equation of state but also in any thermodynamic formula. (In earlier work on the
present subject this proposition had been regarded as an assumption.) Thus, the parameters in that part of In $f_{w}$ which arises from the intermolecular forces are the virial coefficients for water vapor, those for air, and the interaction virial coefficients which correspond to groups of molecules containing at least one water molecule and one "air" molecule. Also, the concept of the intermolecular potential for the interaction of a water molecule and an air molecule has a precise meaning.

As the terms in In $f_{w}$ representing the Poynting and Raoult effects can be calculated with ample accuracy by substituting available property values in thermodynamic formulas, experimental data for $f_{w}$ yield information concerning the part of $\ln f_{w}$ which contains the virial and interaction coefficients. In the present work, when available data for the virial coefficients and a sufficient estimate of one of the interaction coefficients are substituted, the experimental data for $f_{w}$ yield data for $B_{a w}(T)$ and $C_{a a w}(T)$, which are, respectively, the interaction second virial coefficient corresponding to the interaction of a water mol ecule with an air molecule and the interaction third virial coefficient corresponding to the interaction of a water molecule with two air molecules. The coefficients $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$ together with those available from other sources form a complete set of second and third virial and interaction virial coefficients for moist air. Subject to limitations of accuracy, this set allows all the thermodynamic properties of the gas mixture to be calculated.

The Present Work. Extensive measurements of $f_{w}$ for water in air have been made for temperatures of 20,50, and $75^{\circ} \mathrm{C}$ and pressures to 14 MPa . An early report on the project (Wylie and Fisher, 1974) described measurements for 20 and $50^{\circ} \mathrm{C}$, and the derivation of values of $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$. It also included a detailed proof that dry air may betreated as a single component. The air was atmospheric, and contained approximately 3 parts in $10^{4}$ by volume of carbon dioxide. The value of 1.8 parts in $10^{4}$ given in the early report is incorrect. In the following, the theory is outlined and the experiments and the analysis of the results are described. The resulting data for $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ and its first and second derivatives, and thosefor $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ which are the first experimental data to be given for this coefficient, are the most accurate available. Some information can be deduced concerning the first derivative of $\mathrm{C}_{\text {aaw }}(\mathrm{T})$, but very little concerning the second.

As the interaction between the dipol e moment of a water molecule and the moment which this induces in an air molecule contributes to the intermol ecular potential a term of the same form as that for the dispersion force, namely, an inverse sixth-power term, a Lennard-J ones (L-J ) (6m ) potential is a good candidate to represent the whole interaction. As the two parameters of such a potential are determined by the values of $\mathrm{B}_{\text {aw }}$ for only two temperatures, the present results for three allow the consistency obtained with the potential to be tested. With an $L-J(6-12)$, an L-J (6-18), or an L-J (6-24) potential, not only is a very high degree of consistency found, but practically the same values of $\mathrm{B}_{\text {aw }}$ are obtained on extrapolation to much lower or higher temperatures. An L-J (6-12) potential is fitted to the three results, and values of $\mathrm{B}_{\mathrm{aw}}$ and its first and second derivatives are tabulated along with their uncertainties for temperatures from -100 to $+200^{\circ} \mathrm{C}$. To predict reliably the temperature dependence of $\mathrm{C}_{\mathrm{aaw}}$ is a complex problem. On the basis of a linear fit, this coefficient and its uncertainty are tabulated for temperatures from 0 to $100^{\circ} \mathrm{C}$.

The factors $g_{w}(T, p)$ and $f_{w}(T, p)$ and their uncertainties can be calculated with the aid of the derived values and
uncertainties of $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ and $\mathrm{C}_{\text {aaw }}(\mathrm{T})$, correlations (considered below) which exist between the uncertainties in these values and that in the air-water-water interaction coefficient $\mathrm{C}_{\text {aww }}$ being taken into account. A skeleton table for temperatures from 0 to $100{ }^{\circ} \mathrm{C}$ and pressures to 15 MPa is given.

Experimental values of $f_{w}$, or of $g_{w}$ from which $f_{w}$ can easily be derived (eq 4), have been given in several publications (Wylie and Fisher, 1974; Pollitzer and Strebel, 1924; Goff and Gratch, 1945; Webster, 1950; Hyland and Wexler, 1973; Hyland, 1975). Values of $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ have been given for the more accurate of these (Wylie and Fisher, 1974; Goff and Gratch, 1945; Hyland and Wexler, 1973), and some such values have been collated to give further values (Wexler et al., 1981). All these essentially experimental values and two theoretical estimates (Mason and Monchick, 1965; Chaddock, 1965) are represented in a comparison with the present $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ below. For $\mathrm{C}_{\mathrm{aaw}}(\mathrm{T})$, only a tenuous theoretical estimate (Mason and Monchick, 1965) is available for comparison.

Some results obtained with air containing nominally 1\% by volume carbon dioxide will be published in a subsequent paper. They imply that the effect on $f_{w}$ of the amount of carbon dioxide usually present in air (0.03-0.04\% by volume) is negligible. Also, analogous measurements of $f_{w}$ for water in pure oxygen have been made for temperatures of 25,50 , and $75^{\circ} \mathrm{C}$ and pressures to 14 MPa . A relatively brief account of the results and their analysis will be published shortly.

Throughout the paper, the term "uncertainty", unless qualified, means the $3 \sigma$ value obtained by summing the squares of its random and systematic parts. In the terminology developed by the International Organization for Standardization (ISO), the uncertainties quoted are expanded uncertainties obtained by multiplying the appropriately estimated standard deviations by a coverage factor of 3.

## Theory

Derivation of the Equation for $\ln \mathbf{f}_{\mathbf{w}}$. The partial thermodynamic potential of the water vapor in the gas mixture $\mu_{\mathrm{w}}$ is equated to that of the water in the condensed phase $\mu_{\mathrm{w}}^{\prime}$, and the corresponding equation for pure water, $\mu_{\mathrm{w}}^{0}=\mu_{\mathrm{w}}^{\mathrm{I}, \mathrm{O}}$, is subtracted:

$$
\begin{equation*}
\mu_{\mathrm{w}}-\mu_{\mathrm{w}}^{\mathrm{o}}=\mu_{\mathrm{w}}^{\mathrm{l}}-\mu_{\mathrm{w}}^{\mathrm{I}, \mathrm{o}} \tag{5}
\end{equation*}
$$

An expression for $\mu_{\mathrm{w}}$ is derived from the virial equation of state of the gas mixture by calculating firstly the partial molar volume of the vapor and then integrating this with respect to the total pressure.

The equation of state may be written as

$$
\begin{align*}
& \frac{p \tilde{V}}{R T}=1+\left(B_{a a} x_{a}^{2}+2 B_{a w} x_{a} x_{w}+B_{w w} x_{w}^{2}\right) \tilde{N}+ \\
& \left(C_{\text {aaa }} x_{a}^{3}+3 C_{a a w} x_{a}^{2} x_{w}+3 C_{a w w} x_{a} x_{w}^{2}+C_{w w w} x_{w}^{3}\right) \tilde{N}^{2}+\ldots \tag{6}
\end{align*}
$$

where $x_{a}=1-x_{w}$ is the mole fraction of air and $\tilde{V}$ is the molar volume of the mixture. The required expression for $\mu_{\mathrm{w}}$ is found to be

$$
\begin{equation*}
\mu_{\mathrm{w}}=\mu_{\mathrm{w}}^{\square}+\mathrm{RT} \ln \left[\mathrm{c}_{\mathrm{w}} \mathrm{RT} / \mathrm{p}^{\square}\right]+\mathrm{RTW} \tag{7}
\end{equation*}
$$

where $\mu_{\mathrm{w}}^{\square}$ is the thermodynamic potential of pure water vapor in a hypothetical ideal-gas state at temperature T and pressure $\mathrm{p}^{\square}$, and

$$
\begin{equation*}
\mathrm{w}=\frac{2 \beta}{\tilde{V}}+\frac{3}{2} \frac{\gamma}{\tilde{v}^{2}}+\frac{4}{3} \frac{\delta}{\tilde{v}^{3}}+\ldots \tag{8}
\end{equation*}
$$

in which

$$
\begin{align*}
& \beta=B_{a w} \mathrm{x}_{\mathrm{a}}+\mathrm{B}_{\mathrm{ww}} \mathrm{x}_{\mathrm{w}} \\
& \gamma=C_{\text {aaw }} x_{a}^{2}+2 C_{a w w} x_{a} x_{w}+C_{w w w} x_{w}{ }^{2}  \tag{9}\\
& \delta=D_{\text {aaaw }} x_{a}{ }^{3}+3 D_{\text {aaww }} x_{a}{ }^{2} x_{w}+3 D_{\text {awww }} x_{a} x_{w}{ }^{2}+D_{\text {wwww }} x_{w}{ }^{3}
\end{align*}
$$

and so on. To obtain $\mu_{w}^{\circ}$, we simply take $x_{a}=0$ in eq 9 , and substitute $c_{w}^{0}$ for $c_{w}$ in eq 7 .

For the condensed phase, we may calculate $\mu_{\mathrm{w}}^{1}-\mu_{\mathrm{w}}^{\mathrm{I}, 0}$ as the change in the potential which occurs when the pressure on pure liquid water is raised from the saturation vapor pressure to $p$, plus that which occurs when the appropriate amount of air dissolves in the liquid. The first part is $\int_{p_{o}^{p}}^{p} \tilde{V}_{w}^{1,0} d p$, where $\tilde{V}_{w}^{1,0}$ is the molar volume of the pure liquid. With the approximation that the dissolved air, of mole fraction $x_{a}^{\prime}$, forms an ideal solution, the marginally significant second part is $-R T x_{a}^{1}$.

Noting that eqs 1 and 4 give

$$
\begin{equation*}
R T \ln f_{w}=R T \ln \left(Z / Z_{w}^{0}\right)+R T \ln \left[\frac{c_{w} R T}{p^{\square}}\right]-R T \ln \left[\frac{c_{w}^{0} R T}{p^{\square}}\right] \tag{10}
\end{equation*}
$$

we obtain on substituting in eq 5

$$
\begin{equation*}
\ln f_{w}=\ln \left(Z / Z_{w}^{o}\right)-\left(W-W^{0}\right)+\frac{1}{R T} \int_{p_{w}^{o}}^{p} \tilde{V}_{w}^{1, o} d p-x_{a}^{l} \tag{11}
\end{equation*}
$$

where $\mathrm{W}^{\circ}$ is the value of W when $\mathrm{x}_{\mathrm{a}}=0$. We then see from eq 4 that $\ln \mathrm{g}_{\mathrm{w}}$ is given by the right-hand side of eq 11 with the first term deleted.

Some Properties of the Equation for $\ln \mathbf{f}_{\mathbf{w}}$. If the interaction virial coefficients and the terms for the Poynting and Raoult effects are taken equal to zero, eq 11 reduces to $f_{w}=Z / Z_{w}^{0}$. Thus, when hypothetically there is no interaction between the water and the air, it is $g_{w}$ and not $f_{w}$ which is unity.

As $p$ tends to $p_{w}^{o}$, the terms in $W$ which explicitly involve $B_{w w}$ and $C_{w w w}$ naturally tend to the corresponding terms in $\mathrm{W}^{\circ}$. However, as $\mathrm{x}_{\mathrm{w}} / \tilde{V}$ varies with p only in proportion to $g_{w}$, these terms largely cancel at pressures as high as 3 MPa and substantially even at 14 MPa . Therefore, except at the highest pressures, the effect of $B_{w w}$ and $C_{w w w}$ on $g_{w}$ is very small, and their effect on $f_{w}$ occurs largely through the term $\ln \left(Z / Z_{w}^{0}\right)$. Further, except at very low pressures, $Z$ is almost equal to the compressibility factor of dry air. Thus, the self virial coefficients adopted for the separate components must generate the compressibility factors of the components accuately rather than be true virial coefficients.

Ewald, J epson, and Rowlinson (1953), and then other authors, have simplified eq 11 subject to $x_{w} \ll 1$. The only terms of W which they find remain are those in $\mathrm{B}_{\text {aw }}, \mathrm{C}_{\text {aaw }}$, $\mathrm{D}_{\text {aaaw, }}$, and so on. To show that the result is very misleading, we consider the ratio of the term in $\mathrm{C}_{\text {aww }}$ to that in $\mathrm{B}_{\mathrm{aw}}$ :

$$
\begin{equation*}
\frac{\text { term in } C_{a w w}}{\text { term in } B_{a w}}=\frac{3}{2} \frac{C_{a w w}}{B_{a w}} f_{w}\left[\frac{\mathrm{p}_{\mathrm{w}}^{o}}{\mathrm{ZRT}}\right] \tag{12}
\end{equation*}
$$

If $x_{w}$ is made small by lowering $T$, causing $p_{w}^{0}$ and the ratio in eq 12 to diminish rapidly, the result is valid.


Figure 1. Changes in the $f_{w}$ given by eq 11 for $20^{\circ} \mathrm{C}$ and pressures up to 15 MPa for the indicated changes in $\mathrm{Baw}_{\mathrm{aw}}, \mathrm{C}_{\text {aaw }}$, and $\mathrm{C}_{\text {aww }}$.

However, if $x_{w}$ is reduced by increasing the total pressure, as envisaged by the earlier authors, that ratio slowly increases, mainly through $f_{w}$, and the result is false. This can be seen also by comparing curves included in Figure 1 which, however, is given mainly to exemplify the magnitudes and trends of the interaction contributions to $f_{w}$.

As $f_{w}$ is involved in $x_{w}$ (see eq 3) and therefore in W and $Z$, it slightly affects the right-hand side of eq 11 . However, $f_{w}$ can easily be calculated from the equation by iteration.

## Measurements

Outline of the Method. The principal parts of the apparatus are shown diagrammatically in Figure 2. A number of ancillary devices have been omitted from this. Dry compressed atmospheric air from a bank of highpressure cylinders is passed first through a system in which it is very accurately equilibrated with liquid water at a precisely controlled and measured temperature and pressure. This system is essentially a precision humidity generator. The gas then passes into an analyzer in which the water vapor is absorbed (or in the earlier work, condensed in a low-temperature trap and subsequently transferred to absorption tubes) for weighing. The dry air is col lected at a moderate pressure and al so weighed. Then $f_{w}$ is calculated from

$$
\begin{equation*}
f_{w}=\frac{m_{w} / M_{w}}{m_{a} / M_{a}+m_{w} / M_{w}}\left[\frac{p}{p_{w}^{o}(T)}\right] \tag{13}
\end{equation*}
$$

where $M_{w}$ and $m_{w}$ are, respectively, the molecular weight and observed mass of the water and $M_{a}$ and $m_{a}$ the corresponding quantities for dry air.

Typically, with the gas flowing at the adopted nominal rate of $100 \mathrm{~g} / \mathrm{h}$, a single run occupies several hours, but much additional time is occupied in setting and stabilizing the conditions and carrying out the initial and final weighings. We now give some details of sections of the apparatus and the proving of their efficiencies, of the control and measurement of thetemperature and pressure, and of the weighing of the water and air.

Saturator System. Gas enters the system from a bank of high-pressure gas cylinders through two pressurereducing regulators and an adjustable low-resistance valve. It passes first through a heat-exchanger coil and a main saturator in which it bubbles through water in the interstices between glass beads. Then, in a "postsaturator", it flows through a bed of carborundum grits to remove any residual entrained spray and over wet surfaces where saturation is completed by transfer of the vapor. Finally, it passes through a pressure-reducing needle valve. The


Figure 2. Diagram of the apparatus. A number of ancillary systems and components are not shown.
flow rate, which is not critical, is set by the needle valve and observed by diverting the output through a further valve to a flow meter at atmospheric pressure.

The whol e saturator system is immersed in a water bath contained in a 14 L Dewar vessel of 200 mm internal diameter. For temperature control the bath contains also a specially designed resistance thermometer of very rapid response, an electrical wire heater and a closely associated heat-sink tube for work at $20^{\circ} \mathrm{C}$, and an impeller which circulates the water rapidly around a long vertical partition. Liquid can be circulated through the heat sink from an independent small refrigerated and temperaturecontrolled tank. Platinum resistance thermometers can be inserted. Outside, and close to the Dewar vessel, the gasinlet tube is temperature-controlled by a small electronic unit, while all other tubes, electrical leads, and stems are brought out through a similarly temperature-controlled housing which covers the closure of the bath.

The vessel of the main saturator is a 1 L stainless-steel cylinder (Hoke, type HD1000). This is filled with glass beads 4.5 mm in diameter, and then about halfway with water (approximately 230 mL with the beads initially dry). The gas enters at the bottom through eight 3 mm holes in an inside ring-shaped tube. There are two outlet connections, one leading to the bottom of the postsaturator and the other to the pressure-measuring system.

The vessel of the postsaturator is a specially made stainless-steel cylinder 19 mm in internal diameter and about 180 mm long. In this, the gas passes first through a 35 mm deep bed of 60 mesh carborundum grits held between 400 mesh stainless-steel gauzes, and then through six parallel channels lined with white filter paper. The paper stands in an annular reservoir from which water (about 4 mL ) rises 100 mm by capillarity to the top of the channels. Ancillary systems allow both the main saturator and the postsaturator to be charged with known quantities of water while under pressure, causing a minimum of disturbance.

Both pressure vessels are closed with Dowty seals, which are basically of stainless steel and present only a narrow ring of neoprene to the interior. Indeed, all valves and fittings in the saturator and analyzer systems are of type

316 stainless steel and are designed to avoid dead spaces. Tubing of the same steel, with a bright, smooth inside finish and of 4.8 mm outside and 3.4 mm inside diameter, is used throughout. The heat exchanger consists of a coil of 2.2 m of this. Before assembly, all components have been thoroughly cleaned, the last step being to pass steam, allowing the condensate to drain out.

During development of the main saturator the nature of the gas flow through the water between the beads, and the associated rise in water level, were studied using a fullscale glass model, and the mixing of the water (in relation to lateral heat transfer) was observed by making miniscule injections of a strong solution of potassium permanganate.

The principal impediment to temperature uniformity and a high efficiency of the system is the cooling as water evaporates into the incoming dry gas, but this has not been a difficult problem. The distribution of the temperature in the bath and over the saturator vessels was observed under normal operating conditions using multijunction thermocouples. No differences exceeding 1 mK were found.

To prove the efficiency of the main saturator, watertransfer runs were carried out without the postsaturator for progressively increased charges of water, commencing with the glass beads dry. The resulting approximately exponential plot of the saturation deficit shows that the adopted water charge is 4-5 times greater than necessary. Also, again without the postsaturator, water-transfer runs were carried out both with and without an additional saturator (at the same temperature) inserted before the heat exchanger. The results, for temperatures of $0-40^{\circ} \mathrm{C}$ and pressures to 14 MPa , show that the saturation deficit is unlikely to exceed $0.2 \%$.

It was shown using a small Wilson-type cloud chamber that, although the carborundum filter is associated with an entirely negligible drop in pressure, it removed all aerosol particles (condensation nuclei) from the gas. In considering the efficiency of the post saturator, we note that the carborundum filter must adsorb some water vapor. However, tests with a fast-responding hygrometer (Wylie, 1957) have shown that the effect is too rapid to be significant. A fiber-glass filter, on the other hand, hopelessly retards the attainment of equilibrium. We also note
that, on the bench, with evaporation to the atmosphere, capillarity raises water in the filter paper so that it has a glistening appearance at a height of at least 150 mm , and the vapor-pressure deficit corresponding to this water tension is only 7 parts in $10^{6}$.

Rough calculations based on vapour transfer by diffusion alone show that the saturation deficit for the postsaturator is unlikely to exceed $0.3 \%$. Thus, the deficit for the whole saturator system is estimated not to exceed 7 parts in $10^{6}$ plus $0.3 \%$ of $0.2 \%$, or 13 parts in $10^{6}$, which is entirely negligible.

To provide an overall and ongoing test of the efficiency of both the saturator system and the water collection system, runs have occasionally been made for twice the normal flow rates. They have given results not differing significantly from those for the normal rates.

Measurement of the Saturator Temperature and Pressure. The temperature is controlled with a mediumterm stability of 1 mK by means of an electronic proportioning controller. Utilizing the special fast-responding resistance thermometer and wire heater, and having the advantage of the rapid stirring, this corrects small disturbances in seconds. A continuous record of the temperature is provided by a platinum resistance thermometer connected to an automatic resistance bridge with an output to a chart recorder. Variations of 0.2 mK can easily be observed. Also, periodic measurements are made with an ITS-grade platinum resistance thermometer calibrated and used in conjunction with a premium NSL bridge (Thompson and Small, 1971).

It makes no significant difference whether the values of temperature in this paper are regarded as expressed on the IPTS-68 scale or the ITS-90 scale, the difference between which rises monotonically from zero at $0^{\circ} \mathrm{C}$ to 26 mK at $100^{\circ} \mathrm{C}$ (Preston-Thomas, 1990), but temperatures substituted in the adopted formulation for the saturation vapor pressure of water should be on the appropriate scale. The IPTS-68 has been used.

The uncertainty in the measured temperature is conservatively estimated to be 5 mK (systematic part 3 mK ). However, it will be seen below that a constant small error in any one of the experimental temperatures simply produces an obvious offset in the corresponding values of In $f_{w}$, which can be discounted.

The desired operating pressure is set and very accurately controlled by a dead-weight loaded ball-and-cylinder balance through which gas is bled from the input of the saturator system to the atmosphere. Originally devel oped for other purposes, the balance consists of a tungsten carbide ball seated on the plane end of a vertical thickwalled cylinder of nickel-maraging steel, the appropriate load being applied to the ball through a stirrup. The 19 mm ball and the plane square end and 12.7 mm bore of the cylinder were precision formed by lapping.

As the dew point of the gas from the main saturator is close to or above room temperature, a special device is necessary to avoid condensation in the pressure-measuring system. The pressure measurement tube from this vessel and another from the input to the saturator system and containing a small and adjustable flow resistance are taken to a common point from which gas is bled to the atmosphere at a slow rate. The humidity of the released gas is monitored with a CEC Coulometric hygrometer. The flow resistance is adjusted until the humidity indicates that the flows in the two legs are roughly equal. The pressure gauges and balance are connected to the leg containing dry gas, in which the pressure then very dosely approximates that in the saturator.

The pressure is monitored with Wallace and Tiernan gauges of types FA233 and FA234 for pressures to 3.5 MPa, and a series of Heise test gauges from that pressure to 14 MPa. All have scales divided to $0.1 \%$ of full-scale deflection and are used only at high readings. Absolute pressure measurements are made with a Barnet Model V oil-type Iaboratory calibrator which has been calibrated at BemetelTNO and at NPL. The weights used with it have also been calibrated in our own laboratory. All necessary calibration and buoyancy corrections, including those associated with the gas-oil interface are, of course, applied. Atmospheric pressure is monitored with a Lambrecht barograph calibrated periodically with a F ortin barometer. The uncertainty in the measured absolute pressure is conservatively estimated to be 1 part in $10^{4}$ at 1.5 MPa , declining to 0.5 part in $10^{4}$ at 14 MPa , and to be about equally random and systematic.

Anal yzer System. The pressure in this system is about 1.5 MPa . F or the work at 20 and $50^{\circ} \mathrm{C}$ the water was first condensed in a cold trap and then, at the end of a run, transferred slowly into conventional chemical absorption tubes. However, we will describe only the system developed to handle greater concentrations of water vapor which has been used for $75^{\circ} \mathrm{C}$ and all subsequent work, including that with $\mathrm{CO}_{2}$-doped air and that with oxygen. The two systems give the same results and uncertainty in $\mathrm{m}_{\mathrm{w}}$.

The water is absorbed directly in a train of four specially designed cylindrical glass vessels each of 32 mm inside diameter and 130 mL useful volume. The first is charged with anhydrous magnesium perchlorate, the next two with phosphorus pentoxide, and the fourth, used as a tare in weighing, partly with glass beads. Each tube is mounted within a close-fitting metal cylinder, the space between the glass and the metal being pressurized with gas to reduce the pressure difference across the glass to a low level.

The tubes are designed to be weighed on a 100 g balance by stringent classical procedures, their interiors accurately at atmospheric pressure. A buoyancy correction must be applied for the volume increase of the magnesium perchlorate on hydration. The predominant product appears to be the hexahydrate. With some doubt also about the densities of the anhydrate and hexahydrate (Handbook of Chemistry and Physics, 1991-2; Handbook of Chemistry, 1961), the vol ume occupied by the absorbed water has been taken to be $(0.7 \pm 0.2) \mathrm{cm}^{3} / \mathrm{g}$, the corresponding correction to $m_{w}$ being $(0.084 \pm 0.024) \%$.

The efficiency of the system is self-evident in that no significant increase in weight is observed in the third tube. The uncertainty in a measurement of $m_{w}$ is estimated to comprise a random part of 0.16 mg and systematic part of $2.4 \times 10^{-4} \mathrm{~m}_{\mathrm{w}}$.

The dry gas from the water extraction system is collected for weighing in a thin-walled stainless-steel aircraft breath-ing-oxygen cylinder of 35 L capacity and 8.3 kg tare weight. Two similar cylinders serve as tares in the weighing operations, in which buoyancy aspects are a major consideration. An allowance for an increase of approximately 60 mL in cylinder volume at the collection pressure (nominally 1.5 MPa ) is marginally significant. The uncertainty in a measurement of $\mathrm{m}_{\mathrm{a}}$, which is on the order of 600 g , is estimated to consist of a random component of 2.5 parts in $10^{4}$ and an approximately equal systematic component.

Results for $\mathbf{f}_{w}$. Table 1 gives the results for $f_{w}$ averaged for each set of conditions after adjusting the individual results to correspond accurately to the nominal conditions. The scatter of the so adjusted individual results may be seen in Figure 3, referred to further below.

Table 1. Experimental Values for the Vapor-Pressure (or Mole-Fraction) Enhancement Factor $f_{w}$ for Water in Air ${ }^{\text {a }}$

| $\mathrm{p} / \mathrm{MPa}$ | $\mathrm{t}=20^{\circ} \mathrm{C}$ | $\mathrm{t}=50^{\circ} \mathrm{C}$ | $\mathrm{t}=75^{\circ} \mathrm{C}$ |
| ---: | :---: | :---: | :---: |
| 1.4750 | 1.0467 | 1.0394 |  |
| 2.1580 |  |  |  |
| 3.5350 | 1.1144 | 1.0905 | 1.0799 |
| 7.0000 | 1.2356 | 1.1818 |  |
| 7.3480 |  |  |  |
| 10.1000 | 1.3532 | 1.2679 | 1.1668 |
| 14.0750 | 1.5135 | 1.3794 | 1.3165 |

${ }^{\text {a }}$ Each value is the mean of, on the average, six results.


Figure 3. Deviations of the experimental values of $f_{w}$ from the values given by eq 11 with $\mathrm{B}_{\text {aw }}$ and $\mathrm{C}_{\text {aaw }}$ fitted for each temperature (Table 2).

## Property Values Substituted

To obtain $f_{w}$ for each run from eq 13 and analyze the results using eq 11, we require data for $M_{a}, M_{w}, p_{w}^{o}(T)$, $B_{w w}(T), C_{w w w}(T), C_{a w w}(T), \tilde{V}_{w}^{1,0}(T, p)$, and $x_{a}^{\prime}(T, p)$, and also the properties of dry air needed to calculate $Z(T, p)$ and hence $V(T, p)$. Given by Harrison (1965), the molecular weight of dry air containing 3 parts in $10^{4}$ by volume of carbon dioxide is $28.965 \mathrm{~g} / \mathrm{mol}$ ( $\mathrm{C}^{12}$ scale), and that of water $18.0153 \mathrm{~g} / \mathrm{mol}$. The corresponding gas constant is 8.3144 $\mathrm{J} \mathrm{K}^{-1} \mathrm{~mol}^{-1}$. For $\mathrm{p}_{\mathrm{w}}^{0}$ we have adopted the formulation of Wexler (1976), with an uncertainty not exceeding 1 part in $10^{4}$ for temperatures from 0 to $100{ }^{\circ} \mathrm{C}$.

Primarily, $\mathrm{B}_{\mathrm{ww}}$ and $\mathrm{C}_{\text {www }}$ must accurately reproduce the compressibility factor $Z_{w}^{0}$. The data of Wexler et al. (1981) and those of Goff (1949) give the same $Z_{w}^{0}$ within 2 parts in $10^{5}$ at $20^{\circ} \mathrm{C}, 5$ parts in $10^{5}$ at $50^{\circ} \mathrm{C}$, and 1 part in $10^{5}$ at $75^{\circ} \mathrm{C}$, and are equivalent for the present purposes. For temperatures from 0 to $100^{\circ} \mathrm{C}$, the estimated uncertainty in $Z_{w}^{\circ}$ is 4 parts in $10^{4}$. For $\mathrm{C}_{\text {aww }}$ there is no alternative to the theoretical estimate of Hyland and M ason (1975), with an indicated uncertainty of 200\%.

The molar volume $\tilde{V}_{w}^{1,0}(T, p)$ of pure liquid water is taken from Kell (1975) and Kell and Whalley (1965). The first
reference gives the volume at standard atmospheric pressure, and the second the volumefor other pressures relative to that. The uncertainty is negligible. The mole fraction $x_{a}^{1}$ of the dissolved air is the sum of the mole fractions of dissol ved nitrogen, oxygen, and argon. We may lump the argon (about 0.9\%) with the oxygen, as these gases have rather similar intermolecular forces, and then take the partial pressures of the oxygen and nitrogen in the gas to be 0.22 p and 0.78 p, respectively. The low-pressure solubilities have been given as a function of temperature by Himmelblau (1960). Zoss \& al. (1954) have shown that in the present range the solubility of oxygen is proportional to its partial pressure. The solubility of nitrogen, however, is proportional for only about 10\% of the range. From the results of Wiebe et al. (1933) we deduce the adequate approximation that the nonlinearity is independent of the temperature, and represent it by a factor of $1-1.03 \times$ $10^{-2} p_{n} / \mathrm{MPa}$, where $p_{n}$ is the partial pressure of the nitrogen. There is no alternative but to regard the solubilities of the two components as additive. However, $x_{a}^{1}$ is only marginally significant, its greatest value in the present work being 0.0019 which occurs for $20^{\circ} \mathrm{C}$ and 14 MPa .

To calculate $Z(=P \tilde{V} / R T)$, we require firstly all the virial and interaction coefficients of eq 6 . Our source of $B_{w w}$ and $\mathrm{C}_{\text {www }}$ has been given above, and $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$ are products of the present work (an iteration being implied). However, the self virial coefficients of dry air cannot be truncated after the third. The PVT data of Michels et al. (1954), which are undoubtedly more accurate than those of Holborn and Otto (1925), have been adopted. Z has been expressed as a polynomial in $\tilde{V}^{-1}$ which extends to the term in $\tilde{V}^{-7}$. Only the coefficients of the first two terms approximate true virial coefficients, but all the coefficients have been treated as such. The resulting $Z$ is accurate at low pressures, where only the terms in $\tilde{\mathrm{V}}^{-1}$ and $\tilde{\mathrm{V}}^{-2}$ are significant. It is also accurate at high pressures, where terms containing $x_{w}$ are negligible and it becomes simply the compressibility factor for dry air. It can further be shown to be accurate in the transition region.

The Michels results for air already include data for 50 and $75^{\circ} \mathrm{C}$. Values of the seven polynomial coefficients for $20{ }^{\circ} \mathrm{C}$ have been obtained by Bessel's interpolations of coefficients derived from the Michels data for $0,25,50$, and $75{ }^{\circ} \mathrm{C}$. For the present temperatures the uncertainty in $Z$, and hence that in $\tilde{V}$, is estimated to rise in proportion to the pressure to 10 parts in $10^{4}$ at 14 MPa .

## Derivation of $B_{\text {aw }}$ and $C_{\text {aaw }}$

Values of $B_{a w}$ and $C_{a a w}$ for the Experimental Temperatures. The changes in the calculated $\ln f_{w}$ for increments in the $\mathrm{B}_{\text {aw }}$ and $\mathrm{C}_{\text {aaw }}$ substituted in eq 11 are almost proportional to $p$ and $p^{2}$, respectively. Thus, when for a given temperature, approximate values of those coefficients are substituted, and the differences of the cal culated values of $\ln f_{w}$ from the experimental values are least-squaresfitted by a quadratic polynomial in $p$, the coefficients of $p$ and $p^{2}$ give adjustments which should be made to $B_{a w}$ and $\mathrm{C}_{\text {aaw }}$. A single iteration is likely to be adequate. The constant term obtained in the fit, which we denote by a, ideally should be zero, but inevitably a small nonzero value is found. As well as giving a, $\mathrm{B}_{\mathrm{aw}}$, and $\mathrm{C}_{\text {aaw }}$, the final leastsquares fit gives their random uncertainties and the correlations between them.

The values found for $\mathrm{a}, \mathrm{B}_{\mathrm{aw}}$, and $\mathrm{C}_{\mathrm{aaw}}$, and for their random, systematic and total uncertainties, are given in Table 2. The deviations for the final fits are as plotted in Figure 3. For each temperature, the random uncertainty

Table 2. Values of $\mathrm{a}, \mathrm{B}_{\mathrm{aw}}$, and $\mathrm{C}_{\text {aaw }}$ and Their $\mathbf{3} \sigma$ Uncertainties for the Experimental Temperatures

| t/ ${ }^{\circ} \mathrm{C}$ | $10^{4} \mathrm{a}$ | $10^{4} \times$ uncertainty |  |  | $\mathrm{Baw}^{\text {a }}$ (cm ${ }^{3} / \mathrm{mol}$ ) | uncertainty/(cm ${ }^{3} / \mathrm{mol}$ ) |  |  | $\mathrm{C}_{\text {aaw }} /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | uncertainty/(cm $\left.{ }^{3} / \mathrm{mol}\right)^{2}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | rand | syst | total |  | rand | syst | total |  | rand | syst | total |
| 20 | -4 | 15 | 6 | 16 | -32.39 | 0.59 | 0.38 | 0.70 | 1302 | 119 | 42 | 126 |
| 50 | -3 | 16 | 6 | 17 | -24.31 | 0.79 | 0.99 | 1.27 | 1348 | 200 | 80 | 220 |
| 75 | -5 | 14 | 6 | 15 | -18.67 | 0.64 | 1.85 | 1.96 | 1263 | 154 | 112 | 191 |

in $-\mathrm{B}_{\mathrm{aw}}$ is positively correlated with that in $\mathrm{C}_{\text {aaw }}$ with a correlation coefficient of 0.98 . The random uncertainties for the different temperatures are, of course, uncorrelated. Conservatively, the systematic uncertainties for the different temperatures are regarded as fully correlated.

The systematic uncertainties in $\mathrm{a}, \mathrm{B}_{\mathrm{aw}}$, and $\mathrm{C}_{\mathrm{aaw}}$ are the statistical sums of the uncertainties in eq 11 which are, respectively, independent of p , proportional to p , and proportional to $p^{2}$. For a, contributions arise mainly from $Z_{w}^{0}, m_{w}, m_{a}$, and $p_{w}^{0}(T)$ through the measurement of $T$. Possible contributions from a small departure of the air from the accepted composition (affecting mainly $\mathrm{Ma}_{\mathrm{a}}$ ) and the release of adsorbed air from the porous magnesium perchlorate when it is hydrated cannot be estimated and have been neglected (but see next paragraph). For $\mathrm{B}_{\mathrm{aw}}$, the main contribution is from $\mathrm{C}_{\text {aww }}$, while a contribution roughly 10 times smaller arises from $Z$. For $\mathrm{C}_{\text {aaw, }}$, a dominant contribution arises from $\mathrm{C}_{\text {aww }}$. The contributions from this coefficient are easily calculated because the dependence of the $C_{\text {aww }}$ term on the pressure can be resolved without significant residue into a component proportional to $p$ and one proportional to $p^{2}$ by leastsquares fitting.

That a, which varies surprisingly little from temperature to temperature, is smaller than even its systematic uncertainty $\left(6 \times 10^{-4}\right)$ testifies to the high absolute accuracy of the work. However, it will be seen below that its value has no effect on the values of $B_{a w}, C_{a a w}, g_{w}$, and $f_{w}$ derived.

Values of $B_{\text {aw }}$ and Its Derivatives for a Range of Temperatures. The two parameters of a Lennard-J ones potential can readily be derived from the values of $\mathrm{B}_{\text {aw }}$ for two temperatures using formulas given, for example, by Hirschfelder et al. (1964). To interpolate and extrapolate our data for $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$, we use a least-squares method to fit an L-J (6-12) potential to our values for three temperatures. (It makes little difference whether we work with the deviations of $B_{a w}$ or those of $f_{w}$.) In the notation of Hirschfelder et al., the result is

$$
\begin{gather*}
\epsilon / \mathrm{K}=140.67 \mathrm{~K} \\
\mathrm{~b}_{\mathrm{o}}=57.61 \mathrm{~cm}^{3} / \mathrm{mol} \tag{14}
\end{gather*}
$$

Corresponding values of $\mathrm{B}_{\mathrm{aw}}, \mathrm{T}\left(\mathrm{dB}_{\mathrm{aw}} / \mathrm{dT}\right)$, and $\mathrm{T}^{2}\left(\mathrm{~d}^{2} \mathrm{~B}_{\mathrm{aw}} /\right.$ $\mathrm{dT}^{2}$ ) and their uncertainties are given in Table 3 for temperatures of -100 to $+200^{\circ} \mathrm{C}$. The deviations of the experimental $\mathrm{B}_{\mathrm{aw}}$ for 20,50 , and $75^{\circ} \mathrm{C}$ are, respectively, only $0.01,-0.04$, and $0.05 \mathrm{~cm}^{3} / \mathrm{mol}$, much smaller than even the corresponding random uncertainties of $0.59,0.79$, and $0.64 \mathrm{~cm}^{3} / \mathrm{mol}$.

When an L-J (6-18) or an L-J (6-24) potential is fitted instead, the changes in Table 3 are negligible except at the lowest temperatures. Even with the latter potential, the change in $\mathrm{B}_{\text {aw }}$ at $-50{ }^{\circ} \mathrm{C}$ is only $-0.7 \mathrm{~cm}^{3} / \mathrm{mol}$ compared with an uncertainty of $3.4 \mathrm{~cm}^{3} / \mathrm{mol}$, and at $-100{ }^{\circ} \mathrm{C}$ only $-3.8 \mathrm{~cm}^{3} / \mathrm{mol}$ compared with $7.6 \mathrm{~cm}^{3} / \mathrm{mol}$. This insensitivity to the form of the potential accords with the similar property for pure substances (Maitland et al., 1981; Rowlinson et al., 1954).

Treatment of the Results for $\mathbf{C}_{\text {aaw. }}$. A reliable theoretical interpolation of $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ would be a major undertak-
ing, especially as a polar molecule is involved. Without it, the data (Table 2) warrant no more than a weighted linear least-squares fit. This gives the equation

$$
\begin{equation*}
\mathrm{C}_{\mathrm{aaw}}(\mathrm{~T}) /\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}=1320-0.47 \mathrm{t} /{ }^{\circ} \mathrm{C} \tag{15}
\end{equation*}
$$

Its relationship to the experimental points may be seen in Figure 6.

On a wide scale, the $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ curve must have a maximum at which $\mathrm{C}_{\text {aaw }}$ is positive, below which it falls with rapid acceleration to negative values, and above which it passes through an inflexion and then asymptotically approaches zero. Our experimental range clearly does not lie at temperatures below the maximum, and with extrapolation to lower temperatures the uncertainty must soon increase rapidly.

Values of $\mathrm{C}_{\text {aaw }}$ obtained from eq 15 for temperatures from 0 to $100^{\circ} \mathrm{C}$ are given with their uncertainties in Table 4. In addition to random and systematic components of uncertainty arising from those in the three fitted values (Table 2), the uncertainties in Table 4 include an estimated statistically independent component to allow for our limited knowledge of the true shape of the $\mathrm{C}_{\text {aaw }}(\mathrm{T})$ curve in the relevant temperature range. This amounts, respectively, to 10,55 , and $25\left(\mathrm{~cm}^{3} / \mathrm{mol}\right)^{2}$ at 20,50 , and $75^{\circ} \mathrm{C}$, and increases to $100\left(\mathrm{~cm}^{3} / \mathrm{mol}\right)^{2}$ at $0{ }^{\circ} \mathrm{C}$ and $75\left(\mathrm{~cm}^{3} / \mathrm{mol}\right)^{2}$ at $100^{\circ} \mathrm{C}$.

Correlations of the Uncertainties in $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$. When values of $\mathrm{B}_{\text {aw }}$ and $\mathrm{C}_{\text {aaw }}$ are obtained for an arbitrary temperature to calculate $g_{w}$ or $f_{w}$, it is necessary to know the correlations between their uncertainties, and between those uncertainties and that of $\mathrm{C}_{\text {aww }}$, to calculate the uncertainty in the result. The random uncertainties in values of $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$ for such a temperature do not have the almost $100 \%$ correlation of those in Table 2. That in $\mathrm{B}_{\text {aw }}$ has three components attributable to those of the three values in that table. Similarly, the systematic uncertainty has three components. The random and systematic uncertainties in $\mathrm{C}_{\text {aaw }}$ each has the analogous three components, but in addition $\mathrm{C}_{\text {aaw }}$ has the component which allows for the uncertainty concerning the shape of the $\mathrm{C}_{\text {aaw }}$ curve. Random components in $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$ which arise from measurements at different temperatures are uncorrelated, but those which relate to the same temperature are practically 100\% correlated.

As all the systematic uncertainties in Table 2 arise very largely from the uncertainty in $\mathrm{C}_{\text {aww }}$, the error in which may be regarded as roughly constant over the present temperature range, there are six systematic components of uncertainty in $\mathrm{B}_{\mathrm{aw}}$ and $\mathrm{C}_{\text {aaw }}$ which are regarded as practically 100\% correlated with one another and with the uncertainty in $\mathrm{C}_{\text {aww }}$. The shape component in $\mathrm{C}_{\text {aaw }}$ is statistically independent.

## Calculations of $\mathbf{g}_{\mathbf{w}}$ and $\mathbf{f}_{\mathbf{w}}$

From eqs 11 and 4 we have

$$
\begin{equation*}
\ln g_{w}=-\left(W-W^{0}\right)+\frac{1}{R T} \int_{p_{w}^{p}}^{p} \tilde{v}_{w}^{1, o} d p-x_{a}^{1} \tag{16}
\end{equation*}
$$

Regardless of the errors in the property values substituted,

Table 3. Values of $\mathbf{3} \sigma$ Uncertainties of $B_{a w}, T\left(d B_{a w} / d T\right)$, and $T^{2}\left(d^{2} B_{a w} / d^{2}\right)$ for Temperatures from $-\mathbf{1 0 0}$ to $+\mathbf{2 0 0}{ }^{\circ} \mathrm{C}$

| t/ ${ }^{\circ} \mathrm{C}$ | $\begin{gathered} \mathrm{Baw} / \\ \left(\mathrm{cm}^{3} / \mathrm{mol}\right) \end{gathered}$ | uncertainty/ (cm³/mol) | $\begin{aligned} & \hline \mathrm{T}\left(\mathrm{~dB}_{\mathrm{aw}} / \mathrm{dT}\right) / \\ & \left(\mathrm{cm}^{3} / \mathrm{mol}\right) \end{aligned}$ | uncertainty/ (cm ${ }^{3} / \mathrm{mol}$ ) | $\begin{gathered} \mathrm{T}^{2}\left(\mathrm{~d}^{2} \mathrm{Baw}_{\mathrm{aw}} / \mathrm{dT}^{2}\right) / \\ \left(\mathrm{cm}^{3} / \mathrm{mol}\right) \end{gathered}$ | uncertainty/ (cm ${ }^{3} / \mathrm{mol}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -100 | -101.0 | 7.6 | 185.1 | 19.2 | -461 | 40 |
| -90 | -91.0 | 6.6 | 170.4 | 18.1 | -420 | 38 |
| -80 | -82.3 | 5.6 | 157.7 | 17.0 | -385 | 36 |
| -70 | -74.6 | 4.8 | 146.7 | 16.1 | -355 | 34 |
| -60 | -67.8 | 4.1 | 137.1 | 15.2 | -330 | 33 |
| -50 | -61.7 | 3.4 | 128.6 | 14.5 | -307 | 31 |
| -40 | -56.2 | 2.8 | 121.1 | 13.7 | -288 | 30 |
| -30 | -51.3 | 2.2 | 114.3 | 13.1 | -270 | 28 |
| -20 | -46.81 | 1.76 | 108.2 | 12.5 | -255 | 27 |
| -10 | -42.72 | 1.33 | 102.7 | 11.9 | -241 | 26 |
| 0 | -38.99 | 0.98 | 97.7 | 11.4 | -228 | 25 |
| 5 | -37.24 | 0.84 | 95.3 | 11.2 | -222 | 24 |
| 10 | -35.56 | 0.74 | 93.1 | 10.9 | -217 | 24 |
| 15 | -33.95 | 0.68 | 91.0 | 10.7 | -212 | 24 |
| 20 | -32.40 | 0.67 | 88.9 | 10.5 | -207 | 23 |
| 25 | -30.91 | 0.71 | 86.9 | 10.3 | -202 | 23 |
| 30 | -29.48 | 0.78 | 85.1 | 10.1 | -197 | 22 |
| 35 | -28.11 | 0.88 | 83.2 | 9.9 | -193 | 22 |
| 40 | -26.78 | 0.99 | 81.5 | 9.7 | -189 | 21 |
| 45 | -25.50 | 1.10 | 79.8 | 9.5 | -185 | 21 |
| 50 | -24.27 | 1.22 | 78.2 | 9.3 | -181 | 21 |
| 55 | -23.08 | 1.34 | 76.6 | 9.2 | -177 | 20 |
| 60 | -21.94 | 1.46 | 75.1 | 9.0 | -173 | 20 |
| 65 | -20.83 | 1.58 | 73.7 | 8.8 | -170 | 20 |
| 70 | -19.76 | 1.70 | 72.3 | 8.7 | -166.5 | 19.3 |
| 75 | -18.72 | 1.82 | 71.0 | 8.5 | -163.3 | 19.0 |
| 80 | -17.72 | 1.93 | 69.7 | 8.4 | -160.2 | 18.7 |
| 85 | -16.8 | 2.0 | 68.4 | 8.2 | -157.2 | 18.4 |
| 90 | -15.8 | 2.1 | 67.2 | 8.1 | -154.3 | 18.1 |
| 95 | -14.9 | 2.3 | 66.0 | 8.0 | -151.5 | 17.8 |
| 100 | -14.0 | 2.4 | 64.8 | 7.8 | -148.9 | 17.6 |
| 110 | -12.3 | 2.5 | 62.7 | 7.6 | -143.8 | 17.0 |
| 120 | -10.7 | 2.7 | 60.6 | 7.3 | -139.0 | 16.5 |
| 130 | -9.2 | 2.9 | 58.7 | 7.1 | -134.5 | 16.1 |
| 140 | -7.8 | 3.1 | 56.8 | 6.9 | -130.2 | 15.6 |
| 150 | -6.5 | 3.2 | 55.1 | 6.7 | -126.3 | 15.2 |
| 160 | -5.2 | 3.4 | 53.5 | 6.5 | -122.5 | 14.8 |
| 170 | -4.0 | 3.5 | 51.9 | 6.3 | -118.9 | 14.4 |
| 180 | -2.9 | 3.7 | 50.5 | 6.1 | -115.6 | 14.6 |
| 190 | -1.8 | 3.8 | 49.1 | 5.9 | -112.4 | 13.7 |
| 200 | -0.8 | 3.9 | 47.7 | 5.8 | -109.4 | 13.6 |

Table 4. Values and $3 \boldsymbol{\sigma}$ Uncertainties of $\mathrm{C}_{\text {aaw }}$ for Temperatures from 0 to $100^{\circ} \mathrm{C}$

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{C}_{\text {aaw/ }}$ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | uncertainty/ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}\right)^{2}$ | $\mathrm{C}_{\text {aaw }} /{ }^{\circ} \mathrm{C}$ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}^{2}\right)^{2}$ | uncertainty/ $/$ <br> $\left(\mathrm{cm}^{3} / \mathrm{mol}^{2}\right.$ |  |
| :---: | :---: | :---: | ---: | :---: | :---: |
| 0 | 1320 | 200 | 60 | 1290 | 150 |
| 10 | 1320 | 150 | 70 | 1290 | 170 |
| 20 | 1310 | 120 | 80 | 1280 | 200 |
| 30 | 1310 | 110 | 90 | 1280 | 230 |
| 40 | 1300 | 120 | 100 | 1270 | 270 |
| 50 | 1300 | 130 |  |  |  |

every term in eq 16 tends to zero (and $g_{w}$ approaches unity) as $p$ is decreased to $p_{w}^{\circ}$. There is, therefore, no question of including the offset a, or any part of it, in the equation. Thus $g_{w}$ and its uncertainty may be calculated directly from eq 16. The random uncertainty in $g_{w}$ arises from $B_{a w}$ and $\mathrm{C}_{\text {aaw, }}$ the six contributions from which are correlated as described above. The systematic uncertainty arises largely from $\mathrm{B}_{\text {aw }}, \mathrm{C}_{\text {aaw }}$, and $\mathrm{C}_{\text {aww, }}$, the correlations involved also having been already described.

Having thus obtained $g_{w}$ and its uncertainty, we obtain $f_{w}$ by multiplying $g_{w}$ by $Z / Z_{w}^{0}$, and derive its uncertainty by statistically deleting the uncertainty in Z, which largely cancels with the uncertainty allowed for $Z$ in the analysis which leads to Table 2, and statistically adding-in the uncertainty in $Z_{w}^{0}$, which was discarded in the analysis when a was taken equal to zero.

For a wide range of relatively low total pressures, the uncertainty in $Z_{w}^{0}$, which is independent of the pressure, is important. Thus, for $20^{\circ} \mathrm{C}$ and atmospheric pressure,
for example, the total uncertainty in $\mathrm{g}_{\mathrm{w}}(=1.00337)$ is only 0.5 part in $10^{4}$ but that in $f_{w}(=1.0042)$ is practically the same as in $Z_{w}^{0}$, i.e., 4 parts in $10^{4}$. For a range of conditions, Wexler et al. (1981) erroneously assign to $f_{w}$ an uncertainty well below that which they recognize for $Z_{w}^{0}$.

The averaged experimental values of $f_{w}$ (Table 1) may be compared with values calculated from eq 11, $\mathrm{B}_{\text {aw }}$ being obtained from eq 14, or equivalently Table 3, and $\mathrm{C}_{\text {aaw }}$ from eq 15, equivalently Table 4 . For the fifteen sets of conditions, the rms difference is found to be 8 parts in $10^{4}$. If a is added to the experimental values, this is reduced to 6 parts in $10^{4}$. Almost half of each rms difference is contributed by the point for $50{ }^{\circ} \mathrm{C}$ and 14 MPa , and is associated with the greater deviation of $\mathrm{C}_{\text {aaw }}$ for that temperature seen in Figure 6.

A skeleton table of values of $g_{w}$ and $f_{w}$ and their uncertainties is given for temperatures from 0 to $100{ }^{\circ} \mathrm{C}$ and pressures to 15 MPa in Table 5.

## Discussion

Comparison with Earlier Work. At NBS, Hyland and Wexler (1973) made 5 measurements of $\mathrm{f}_{\mathrm{w}}$ for $30^{\circ} \mathrm{C}, 5$ for $40^{\circ} \mathrm{C}$, and 17 for $50^{\circ} \mathrm{C}$. Hyland (1975) later supplemented these with one measurement for $-20^{\circ} \mathrm{C}$, two for $-10^{\circ} \mathrm{C}$, and four for $70^{\circ} \mathrm{C}$, and then collated all the corresponding values of $B_{a w}(T)$. (Only four of the NBS measurements are for pressures above 5 MPa .) The present $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ is plotted in Figure 4 with Hyland's collated values, the experimental results of Goff and co-workers (1945), and the theoretical

Table 5. Values of $\mathbf{g}_{\mathbf{w}}$ and $\mathbf{f}_{\mathbf{w}}$ for Various Temperatures and Pressures ${ }^{a}$

| $\mathrm{t} /{ }^{\circ} \mathrm{C}$ | $\mathrm{p} / \mathrm{MPa}=0.1$ | $\mathrm{p} / \mathrm{MPa}=1$ | $\mathrm{p} / \mathrm{MPa}=2$ | $\mathrm{p} / \mathrm{MPa}=5$ | $\mathrm{p} / \mathrm{MPa}=10$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $1.00420,0.9$ | $1.0428,9$ | $1.0870,17$ | $1.2272,37$ | $1.4794,63$ | $1.7397,98$ |
|  | $1.00412,4.1$ | $1.0376,9$ | $1.0761,17$ | $1.2003,37$ | $1.4361,63$ | $1.3642,18$ |
| 20 | $1.00333,0.5$ | $1.0343,5$ | $1.0694,8$ | $1.1778,16$ | $1.365,98$ |  |
|  | $1.00417,4.0$ | $1.0321,6$ | $1.0640,9$ | $1.1648,16$ | $1.3494,18$ | $1.5489,20$ |
| 40 | $1.00258,0.3$ | $1.0278,3$ | $1.0560,6$ | $1.1417,14$ | $1.2842,22$ | $1.4524,20$ |
|  | $1.00482,4.0$ | $1.0287,5$ | $1.0555,7$ | $1.1393,14$ | $1.2889,22$ | $1.448,32,32$ |
| 60 | $1.00184,0.3$ | $1.0226,4$ | $1.0457,7$ | $1.1147,15$ | $1.2267,25$ | $1.3317,35$ |
|  | $1.00580,4.0$ | $1.0271,6$ | $1.0503,8$ | $1.1215,15$ | $1.2462,25$ | $1.3770,35$ |
| 80 | $1.00102,0.3$ | $1.0184,5$ | $1.0376,9$ | $1.0944,21$ | $1.1849,34$ | $1.2682,45$ |
|  | $1.00566,4.0$ | $1.0274,6$ | $1.0479,10$ | $1.1101,21$ | $1.2172,34$ | $1.3278,45$ |
| 100 |  |  | $1.0147,7$ | $1.0309,14$ | $1.0786,34$ | $1.1535,64$ |
|  |  |  | $1.0292,8$ | $1.0481,14$ | $1.1039,34$ | $1.1985,64$ |

a Upper values, $g_{w}$; Iower values, $f_{w}$. Uncertainities (3 $\sigma$ ) are parts in $10^{4}$.


Figure 4. $\mathrm{Baw}_{\mathrm{aw}}(\mathrm{T})$ for the present work and earlier work: C , Chaddock (1965); G\&G, Goff and Gratch (1945); H, Hyland (1975); W\&F, Wylie and Fisher (1974 and the present work).
estimates of Mason and Monchick (1965) and Chaddock (1965).

As the Goff measurements, for temperatures from 5 to $25^{\circ} \mathrm{C}$, were restricted to pressures of a few atmospheres so that the interaction third virial coefficients, unknown at the time, could be neglected, it is remarkable that those researchers achieved even the accuracy apparent in Figure 4. Their results agree with ours within their implied $3 \sigma$ uncertainty.

Wexler et al. (1981) later collated all the NBS results for $\mathrm{B}_{\mathrm{aw}}$, our 1974 results, and our early results for $75^{\circ} \mathrm{C}$ (privately communicated), and included the resulting values in an extensive tabulation of derived system properties (Wexler et al., 1981). Figure 5 shows the differences of the respective NBS data and of this collation from the present results. Hyland's collation differs from the present results by 2.0 times the present uncertainty at $0^{\circ} \mathrm{C}$ but becomes equal at about $110^{\circ} \mathrm{C}$. The collation of Wexler et al . (1981) differs by 2.6 times the present uncertainty at 0 ${ }^{\circ} \mathrm{C}$, becoming equal near $80^{\circ} \mathrm{C}$.

Both the theoretical estimates of $\mathrm{B}_{\mathrm{aw}}(\mathrm{T})$ were made using L-J (6-12) potentials, and using rules to combine the two parameters of the air-air potential with the appropriate parameters of the water-water potential. However, while Mason and Monchick used the simplest form of the largely empirical combining rules, Chaddock used a form with more regard for the permanent dipole moment of the water molecule. Figure 4 shows that neither of these simple methods gives a good estimate.


Figure 5. Differences of the NBS data for $\mathrm{B}_{\mathrm{aw}}$ from the present data: (a) Hyland and Wexler (1973); (b) Hyland (1975); (c) Wexler et al. (1981). The broken line shows the $3 \sigma$ total uncertainty in the present results.


Figure 6. Present results for $\mathrm{C}_{\text {aaw }}$ and the theoretical results of Mason and Monchick (1965). The error bars show the $3 \sigma$ total uncertainties calculated independently for each temperature.

The only earlier values of $\mathrm{C}_{\text {aaw }}$ are those obtained theoretically by Mason and Monchick (1965) using the combining procedure of Rowlinson et al. (1954). They are plotted in Figure 6 along with the present values, from which they differ by several times the present uncertainty. However, the difference is embraced by the rather tentative estimate of an uncertainty of a factor of 2 given by those authors. The Mason and $M$ onchick estimate of $C_{\text {aaw }}$, being all that was available, has been used by Wexler and Hyland in all their work.

Some Further Considerations. The present data for $\mathrm{B}_{\mathrm{aw}}$ are comparable in uncertainty with the best data available for self second virial coefficients. The data for $\mathrm{C}_{\text {aaw }}$ are the only experimental data available. Except for low pressures, the correlations between the uncertainties in the respective terms are likely to mean that the accuracy of calculations of $f_{w}, g_{w}$, or other thermodynamic properties would be impaired if data for $\mathrm{B}_{\mathrm{aw}}, \mathrm{C}_{\text {aaw, }}$ or $\mathrm{C}_{\text {aww }}$ from other sources were brought in. This would be likely to remain true even if a much better estimate of $\mathrm{C}_{\text {aww }}$ became
available, unless the present experimental results were reanalyzed to give slightly modified values of $\mathrm{B}_{\text {aw }}$ and $\mathrm{C}_{\text {aaw }}$.

The density series for W (eq 8) can be inverted to a pressure series. Sample calculations have shown that if only the second and third virial coefficients of the density expansion are retained, and W is inverted to the pressure form, it is necessary for the present purposes to retain terms as far as that in $p^{6}$ to preserve the accuracy. Thus, at least in the present context, the density series converges much more rapidly than the pressure series.

## Literature Cited

Chaddock, J. B. Moist Air Properties from Tabulated Virial Coefficients. Humidity and Moisture, Reinhold: New York, 1965; Vol. 3, pp 273-284.
Ewald, A. H.; J epson, W. B.; Rowlinson, J . S. The Solubility of Solids in Gases. Discuss. Faraday Soc. 1953, No. 15, 238-243.
Goff, J. A. Final Report of the Working Subcommittee of the International J oint Committee on Psychrometric Data. Trans. ASME 1949, 71, 903-913.
Goff, J. A.; Gratch, S. Thermodynamic Properties of Moist Air. Heat./ Piping/ Air Cond. (ASHVE J. Sect.) 1945, 17, 334-348.
Handbook of Chemistry, 10th ed.; Lange, N. A., Ed.; McGraw-Hill: New York, 1961; pp 268-269.
Handbook of Chemistry and Physics, 72nd ed.; CRC Press: Boca Raton, FL 1991-92; Section 4, p 71.
Harrison, L. P. Fundamental Concepts and Definitions Relating to Humidity. Humidity and Moisture; Reinhold: New York, 1965; Vol. $3, \mathrm{pp} 3-69$.
Himmelblau, D. M. Solubilities of Inert Gases in $\mathrm{H}_{2} \mathrm{O}$ : $0^{\circ}$ to near the Critical Point of $\mathrm{H}_{2} \mathrm{O}$. J. Chem. Eng. Data 1960, 5, 10-15.
Hirschfelder, J. O.; Curtiss, C. F.; Bird, R. B. Molecular Theory of Gases and Liquids; Wiley: New York, 1964.
Holborn, L.; Otto, J. On the Isotherms of a Gas between +400 and $-183^{\circ} \mathrm{C}$. Z. Phys. 1925, 33, 1-11.
Hyland, R. W. A Correlation for the Second Interaction Virial Coefficients and Enhancement Factors for Moist Air. J. Res. Natl. Bur. Stand. (U.S.) 1975, A79, 551-560.
Hyland, R. W.; Mason, E. A. Corrections to Paper Entitled "Third Virial Coefficient for Air-Water Vapour Mixtures". J. Res. Natl. Bur. Stand. (U.S.) 1975, A79, 775-776.
Hyland, R. W.; Wexler, A. The Enhancement of Water Vapour in Carbon Dioxide-Free Air at 30, 40 and $50^{\circ} \mathrm{C}$. J. Res. Natl. Bur. Stand. (U.S.) 1973, A77, 115-131; v. also A77, 133-147.
Kell, G. S. Density, Thermal Expansivity, and Compressibility of Liquid Water from $0^{\circ}$ to $150^{\circ} \mathrm{C}$ : Correlations and Tables for Atmospheric Pressure and Saturation Reviewed and Expressed on 1968 Temperature Scale. J. Chem. Eng. Data 1975, 20, 97-105.

Kell, G. S.; Whalley, E. The PVT Properties of Water. I. Liquid Water in the Temperature Range 0 to $150^{\circ} \mathrm{C}$ and at Pressures up to 1 kb . Philos Trans. R. Soc. London, A. 1965, 258, 565-614.
Maitland, G. C.; Rigby, M.; Smith, E. B.; Wakeham, W. A. Intermolecular Forces: Their Origin and Determination; Clarendon: Oxford, 1981; p 133; see also their ref 2, p 253.
Mason, E. A.; Monchick, L. Survey of the Equation of State and Transport Properties of Moist Gases. Humidity and Moisture; Reinhold: New York, 1965; Vol. 3, pp 257-272.
Mason, E. A.; Spurling, T. H. TheVirial Equation of State; Pergamon: Oxford, 1969; p 111.
Mayer, J. E. Statistical Mechanics of Condensing Systems. V. TwoComponent Systems. J. Phys. Chem. 1939, 43, 71-95.
Michels, A.; Wassenaar, T.; van Seventer, W. I sotherms of Air between $0^{\circ} \mathrm{C}$ and $75^{\circ} \mathrm{C}$ and at Pressures up to 2200 ATM. Appl. Sci. Res. 1953, A4, 52-56.
Pollitzer, F.; Strebel, E. On the Influence of an Indifferent Gas on the Concentration of the Saturated Vapour of Liquids. Z. Phys. Chem. 1924, 110, 768-785.
Preston-Thomas, H. The International Temperature Scale of 1990 (ITS90). Metrologia 1990, 27, 3-10, 107.

Rowlinson, J. S.; Richardson, M. J. The Solubility of Solids in Compressed Gases. Adv. Chem. Phys. 1959, 2, 85-118.
Rowlinson, J. S.; Sumner, F. H.; Sutton, J. R. The Virial Coefficients of a Gas Mixture. Trans. Faraday Soc. 1954, 50, 1-8.
Thompson, A. M.; Small, G. W. A.C. Bridge for Platinum-Resistance Thermometry. Proc. Inst. Electr. Eng. 1971, 118, 1662-1666.
Webster, T. J. The Effect on Water Vapour Pressure of Superimposed Air Pressure. J. Soc. Chem. Ind., L ondon 1950, 69, 343-346.
Wexler, A. Vapour Pressure Formulation for Water in Range 0 to 100 ${ }^{\circ} \mathrm{C}$. A Revision. J. Res. Natl. Bur. Stand. (U.S.) 1976, A80, 775785.

Wexler, A.; Hyland, R.; Stewart, R. Thermodynamic Properties of Dry Air, Moist Air and Water and SI Psychrometric Charts; ASHRAE: Atlanta, 1981
Wiebe, R.; Gaddy, V. L.; Heins, C. J r. The Solubility of Nitrogen in Water at $50^{\circ}, 75^{\circ}$ and $100^{\circ} \mathrm{C}$ from 25 to 1000 Atmospheres. J. Am. Chem. Soc. 1933, 55, 947-953.
Wylie, R. G. A New Absolute Method of Hygrometry. Aust. J. Phys. 1957, 10, 351-365, 429-453.
Wylie, R. G.; Fisher, R. S. Accurate Determination of the Molecular Interaction in a Gas-Vapour Mixture. Fifth Convention of theRoyal Australian Chemical Institute, Canberra, May 1974, Physical Chemistry Section; RACI: Melbourne, 1974.
Zoss, L. M.; Suciu, S. N.; Sibbitt, W. L. The Solubility of Oxygen in Water. Trans. ASME 1954, 76, 69-71.

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